Blind Adaptive Detection of DS/CDMA Signals on Time-Varying Multipath Channels with Antenna Arrays Using High-Order Statistics

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***Abstract*—A new approach based on multiscale decomposition and higher-order statistics is presented for the simultaneous solution of multiuser interference and time-varying multipath propagation in the uplink of a cellular direct-sequence spread-spectrum code-division multiple-access (DS/CDMA) system. Each channel between the mobile transmitter and the base-station receiver is unknown and arbitrarily varying with time. The optimum filter achieving separation and multipath compensation is time-variant. The typical approach in many multiuser detectors recently proposed is to assume that the channel is almost static (time invariant) and attempt detection according to this model. Slow variations of the channels are then compensated using adaptive algorithms that force the estimates (of the channels or of the separating filters) to be constantly in search of a convergence point. If the channel coefficients variations in time are fast with respect to the convergence time of the adaptive algorithm, significant degradation may result. In this work, we depart from this traditional approach and we investigate new kernels that more accurately can characterize the time-varying nature of the detection problem. As a first step, we show that the super-exponential framework can still be applied in a time-variant environment. Then, we describe a multiresolution decomposition of the filter components, essentially constraining its variations in time to remain within the solution subspace. The resulting algorithm overcomes some of the drawbacks associated with slow convergence and insufficient tracking capability typical of many blind approaches and several nonblind methods based on the slow fading assumption.**

***Index Terms*—Array signal processing, code-division multiple access, higher order statistics, interference suppression, land mobile radio cellular systems.**

I. INTRODUCTION

THE PERFORMANCE of code-division multiple-access (CDMA) systems is severely degraded by frequency-selective multipath radio-frequency propagation. The mitigation of this effect based on the use of multiuser/multiantenna detectors has attracted significant interest. The crucial point of almost all of the proposed methods is based on the assumption that the multipath channels are quasi-static, that is time invariant over the length of the transmitted frame. Slow variations of the channels are then compensated by using adaptive algorithms that ultimately force the estimates (of the channels or of the separating filters) to be constantly in search of a convergence point. If the channel coefficients variations in time are fast with respect to the convergence time of the adaptive algorithm, significant degradation may result. Moreover the critical and rather arbitrary quasi-static assumption of the channel does not hold true in the vast majority of the practical cases, especially in high-mobility environments such as those found in macrocell applications. In this work, we depart from the traditional approach and investigate new methods that more accurately can characterize the time-varying nature of the detection problem, and focus on a multiresolution representation of the separating time-variant filter in each component of its response, elaborating some ideas of [20] (and, in a sense, [4]). We show that the super-exponential equations already presented and derived in [11] and [15] are time varying in presence of a time-varying channel. It is important to warn the reader that the super-exponential scheme in a multichannel setup (described and studied in [11] and [15]) is subject to global convergence problems in some pathological cases. Since experimentation has shown successful behavior of the method in practice and because the main topic of the paper is not the analysis of the convergence of the multichannel super-exponential algorithm, we do not address the issue in the sections that follow. The super-exponential algorithm is iterative in nature and the transformation of the technique into a real-time adaptive scheme places a significant challenge whenever the multipath fading channel cannot be assumed static (which is the case in most wireless macrocell systems). The unknown separating filter time variations are decomposed using optimal unconditional bases [3] such as orthonormal wavelet bases [1], [2]. The wavelet transform (WT) is an atomic decomposition that represents a signal in terms of shifted/dilated versions of a prototype bandpass wavelet function $\psi(t)$ and shifted versions of a low-pass scaling function $\phi(t)$ [1]. Because of its excellent time-frequency localization capability, the WT is able to compress most of the energy of the time variations of the filter in the lower resolution representation of the fading process. This gives an approach that is highly effective in fast fading environments and that overcomes in some sense the problem of slow convergence. The paper is organized as follows. In Section II, we review the system model, and in Section III, we describe the algorithmic approach. In Section IV, we motivate a reasonable reduced dimensional representation of the separating filter, and in Section V, we present a real-time implementation. In Section VI, the results of a simulation

Paper approved by S. B. Gelfand, the Editor for Transmission Systems of the IEEE Communications Society. Manuscript received February 5, 1997; revised December 20, 1998, August 20, 1999, and February 17, 2000.

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Publisher Item Identifier S 0090-6778(00)07526-7.

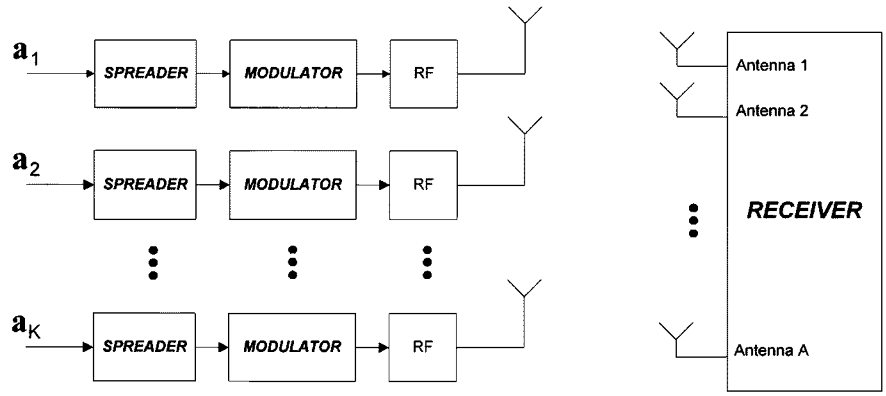

Fig. 1. System model in the uplink of a cellular system.

analysis prove the superiority of the proposed time-variant filter as opposed to the traditionally proposed multiuser minimum mean squared error (MMSE) detector.

II. System Model

In this section, we quickly review the system model in the reverse link of a CDMA cellular system. No particular assumption is made on the geometry of the array at the base station, and we intentionally neglect the propagation assumptions of the model. More details regarding these assumptions can be found in [11], [13], [12]. In a DS/CDMA communication system the information signal relative to the kth user is given by

$$m_k(\tau) = \sum_{n=0}^{M-1} a_k(n) p_{T_s}(\tau - nT_s), \qquad k = 1, 2\ cdots, K \tag{1}$$

where

$$p_{\tilde{\tau}}(\tau) = \begin{cases} 1, & \text{if } 0 < \tau < \tilde{\tau} \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad \mathbf{a} = \{a_k(n)\}_{n=1}^{M}$$

is the message composed of zero-mean identically and independently distributed $\tilde{m}$-ary circularly complex symbols of duration T_s with variance σ_a^2 and fourth-order zero lag cumulant equal to γ_{4a}. We also assume that all odd cumulants of $a_k(n)$ are zero (which is verified by any practical constellation). The K users *asynchronously* share the common channel using the signatures waveforms

$$a_{c,k}(\tau) = \sum_{n=1}^{J} a_{c,n}^k p_{T_c}(\tau - nT_c), \qquad k = 1, 2, \ldots, K \tag{2}$$

where $T_c = (T_s/J)$ is the chip period and $\{a_{c,n}^k \in (-1;\ +1)\}_{n=1}^J$ is the pseudonoise spreading sequence of length J. The spreading signal waveform (2) of the kth user has duration T_s and is normalized to unit power. If an equivalent low-pass representation is employed, the kth transmitted spread-spectrum signal can be written as $\sum_{i=0}^{M-1} a_k(i) p_{T_s}(\tau - iT_s) a_{c,k}(\tau - iT_s)$. This signal is distorted by multipath RF propagation and multiple-access interference caused by other users. Each mobile transmitter has a single antenna (see Fig. 1), while at the base station, the multiplexed signal is received through an A-sensor antenna. The received signal at the base-station receiver can be represented[1] [23], [24], [25], [10] as

$$r^\kappa(t) = \sum_{i=0}^{M-1} \sum_{k=1}^{K} a_k(i) \sqrt{E_k^s(i)} \tilde{f}_k^\kappa(t, t - iT_s) + n^\kappa(t), \quad \kappa = 1, 2, \ldots, A \tag{3}$$

where $\tilde{f}_k^\kappa(t, \tau) = e^{j\theta_k} a_{c,k}(\tau) * \tilde{h}_k^\kappa(t, \tau)$ ($*$ denotes convolution) is the combined impulse response of each signal path of the kth spreading waveform and the channel from the kth user to the κth sensor, $E_k^s(i)$, θ_k are the received signal power and the carrier phase of the kth transmitted signal relative to the kth user, respectively. The channel $\tilde{h}_k^\kappa(t, \tau)$ between the kth transmitter and the κth sensor for $k = 1, 2, \ldots, K$ and $\kappa = 1, 2, \ldots, A$ is

$$\tilde{h}_k^\kappa(t, \tau) = \sum_{l=1}^{Q} f_{k,l}^\kappa(t) \delta(\tau - \tau_{k,l}^\kappa). \tag{4}$$

$f_{k,l}^\kappa(t)$ are the normalized fading complex envelope processes so that $E_k^s(i)$ reflects the kth user received energy over the time period corresponding to iT_s, $\delta(n)$ is the delta function. The noise in (3) is white Gaussian, with two-sided power spectral density $(N_0/2)$; it can also represent the surrounding cell interference, plus noise. Multipath channels enhance interference among users, introducing intersymbol interference and additional correlation between the spreading waveforms. Received energies are assumed invariant over the duration of the message: $E_k^s(i) = E_k^s$ for $i = 0, \ldots, M-1$. We assume that the receiver has perfect knowledge of the powers, time delays $\tau_{k,l}^\kappa$, and phase lags of every received user signal. At the receiver, a bank of KQ filters for each sensor (see Figs. 2 and 3), each filter matched to a delayed replica of the wanted spread-spectrum signature,

[1]Equation (3) with (2) and (1) assumes the code remains the same from bit to bit for a particular user. However IS-95 and many other CDMA systems use different parts of a longer code for each bit. Besides substituting $a_{c,k}^{(i)}(\tau)$, where now the code utilized by the kth user depends on the bit index (i), the rest of the signal model is still valid for systems using long codes (defined aperiodic systems in [9]). For simplicity of exposition we limit our model description to periodic systems. Since the same multichannel model (6) can be derived for the aperiodic (long codes) case the presentation of the algorithms that follow also apply to practical CDMA systems like third-generation (3G)-CDMA and IS-95.

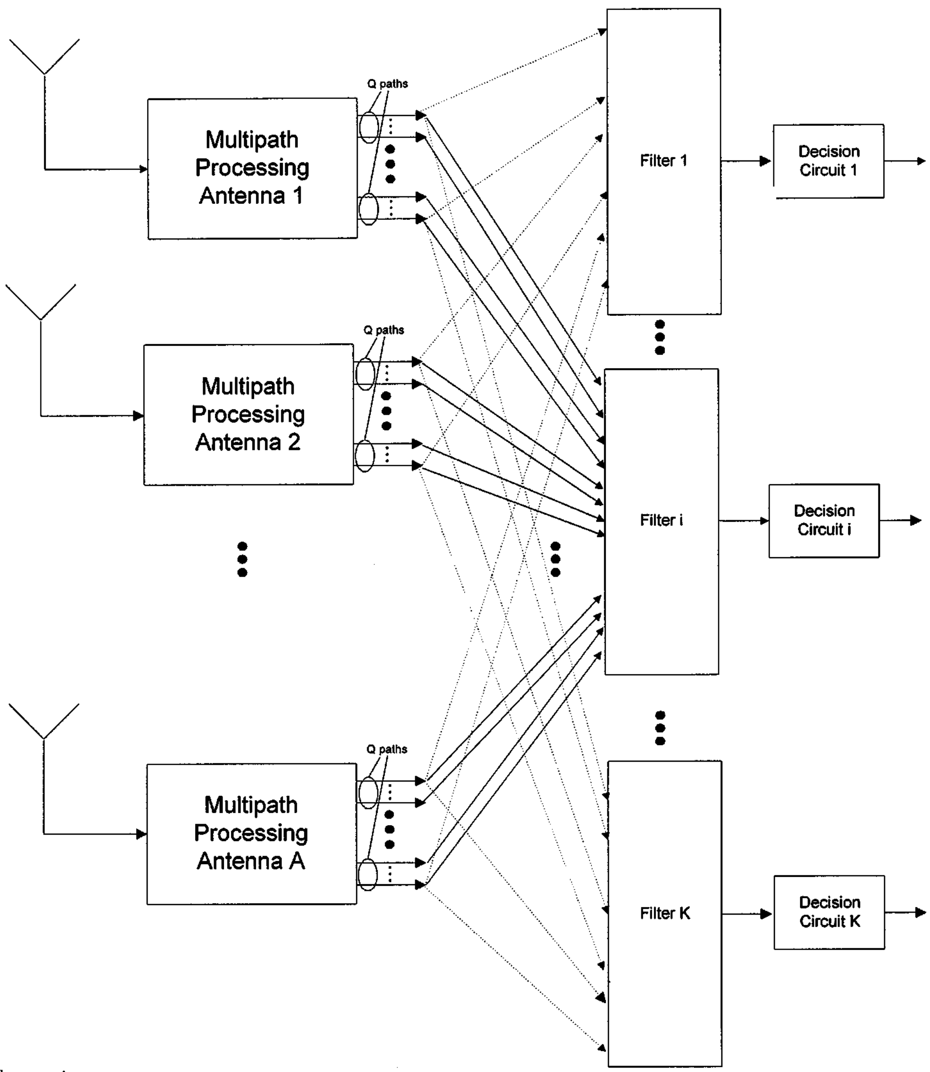

Fig. 2. Architecture of the receiver.

performs despreading with the signatures in (2) and is sampled at symbol rate to obtain the set of samples

$$y^{\kappa}_{k,l}(n) = \int_{nT_s}^{(n+1)T_s} r^{\kappa}(\tau) a_{c,k}(\tau - nT_s - \tau^{\kappa}_{k,l}) d\tau. \tag{5}$$

The input/output relation for the multichannel discrete-time system can be written as

$$\begin{aligned} y^{\kappa}_{i,l}(n) &= \sum_{k=1}^{K}\sum_{q=1}^{Q}\sum_{m} \overline{h}^{\kappa}_{i,k,l,q}(n,m) a_k(n-m) + \eta^{\kappa}_{i,l}(n) \\ &= \sum_{k=1}^{K}\sum_{m} h^{\kappa}_{i,k,l}(n,m) a_k(n-m) + \eta^{\kappa}_{i,l}(n), \\ &\qquad i = 1, 2, \ldots, K, \quad \kappa = 1, 2, \ldots, A, \\ &\qquad l = 1, 2, \ldots, Q \end{aligned} \tag{6}$$

where

$$\begin{aligned} \overline{h}^{\kappa}_{i,k,l,q}(n,m) &= \sqrt{E^s_k} e^{j\theta_k} f^{\kappa}_{k,q}(nT_s) \int_{-\infty}^{+\infty} a_{c,i}(u - \tau^{\kappa}_{i,l}) \\ &\quad \cdot a_{c,k}(u + mT_s - \tau^{\kappa}_{k,q}) du \\ h^{\kappa}_{i,k,l}(n,m) &= \sum_{q=1}^{Q} \overline{h}^{\kappa}_{i,k,l,q}(n,m) \end{aligned} \tag{7}$$

and $\eta^{\kappa}_{i,l}(n)$ is the noise component.

The recovery of the input signals can be achieved by means of a linear KAQ-input K-output time-variant filter (see Fig. 4) that can be represented at time step n in the z-domain as $\mathcal{W}(n,z) = \sum_{k=L_1}^{L_2} \mathbf{W}(n,k) z^{-k}$ with length $L = L_2 - L_1 + 1$. Main objective for $\mathcal{W}(n,z)$ is to achieve *distortionless reception* [11], [12] at any time step n. If we define

$$\begin{aligned} \mathcal{W}(n,z) &= [\tilde{\mathcal{W}}^1(n,z) \quad \tilde{\mathcal{W}}^2(n,z) \quad \cdots \quad \tilde{\mathcal{W}}^A(n,z)] \\ \tilde{\mathcal{W}}^{\kappa}(n,z) &= [\breve{\mathcal{W}}^{\kappa}_1(n,z) \quad \breve{\mathcal{W}}^{\kappa}_2(n,z) \quad \cdots \quad \breve{\mathcal{W}}^{\kappa}_Q(n,z)] \end{aligned}$$

with

$$\breve{\mathcal{W}}^{\kappa}_l(n,z) = \begin{bmatrix} \mathcal{W}^{\kappa}_{1,1,l}(n,z) & \cdots & \mathcal{W}^{\kappa}_{1,K,l}(n,z) \\ \mathcal{W}^{\kappa}_{2,1,l}(n,z) & \cdots & \mathcal{W}^{\kappa}_{2,K,l}(n,z) \\ \vdots & \cdots & \vdots \\ \mathcal{W}^{\kappa}_{K,1,l}(n,z) & \cdots & \mathcal{W}^{\kappa}_{K,K,l}(n,z) \end{bmatrix}$$

distortionless reception means that

$$\mathcal{W}(n,z)\mathcal{H}(n,z) = \mathbf{P}\mathcal{D}(n,z) \tag{8}$$

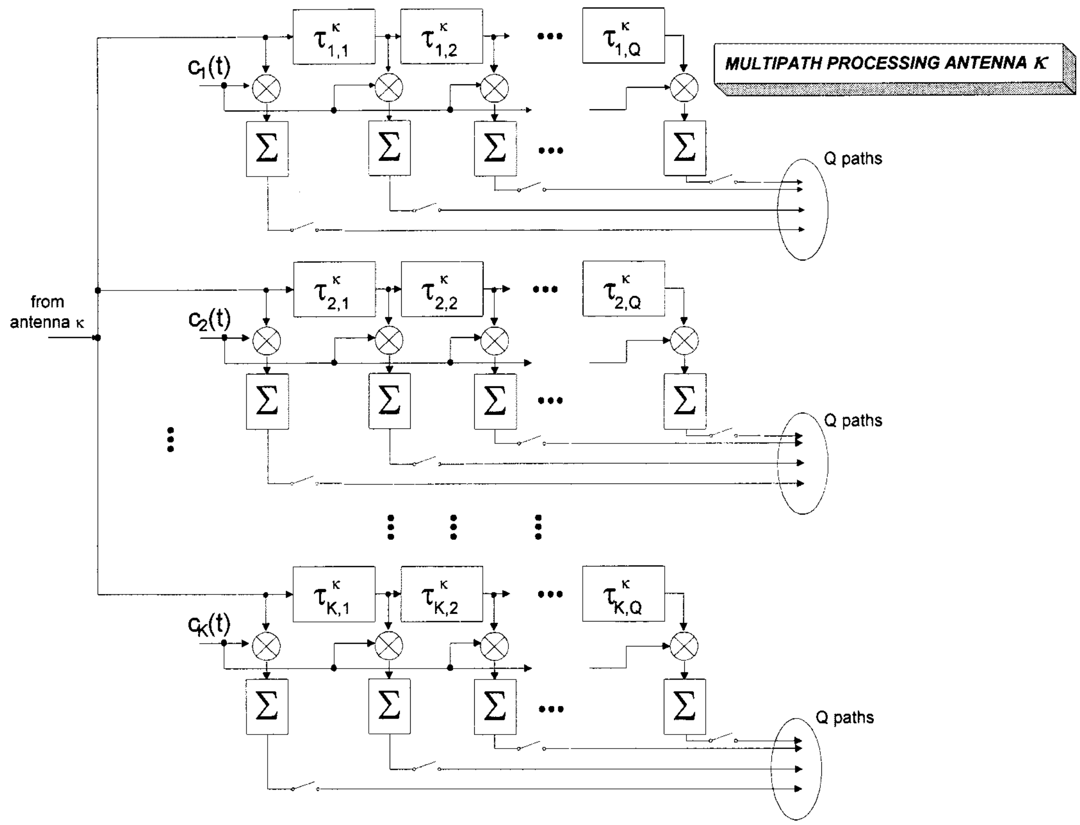

Fig. 3. Signal processing subsection relative to antenna k.

is verified at any n where $\mathbf{P}$ is a permutation matrix

$$\mathcal{D}(n, z) = \mathrm{diag}\{e^{j\phi_1} z^{-n_1}, e^{j\phi_2} z^{-n_2}, \ldots, e^{j\phi_K} z^{-n_K}\}$$

$\phi_i \in [-\pi, \pi] n_i$ integer for $i = 1, 2, \ldots, K$

$$\mathcal{H}(n, z) = \sum_k \mathbf{H}(n, k) z^{-k} \tag{9}$$

and the organization of the $\overline{\mathcal{H}}^{\kappa}_{i,j,l,n}(n, z)$ polynomials (z-transforms of $\overline{h}^{\kappa}_{i,j,l,n}(n, k)$) in $\mathcal{H}(n, z)$ is given by

$$\mathcal{H}(n, z) = \begin{bmatrix} \tilde{\mathcal{H}}^1(n, z) \\ \tilde{\mathcal{H}}^2(n, z) \\ \vdots \\ \tilde{\mathcal{H}}^A(n, z) \end{bmatrix}, \qquad \tilde{\mathcal{H}}^{\kappa}(n, z) = \begin{bmatrix} \check{\mathcal{H}}^{\kappa}_1(n, z) \\ \check{\mathcal{H}}^{\kappa}_2(n, z) \\ \cdots \\ \check{\mathcal{H}}^{\kappa}_Q(n, z) \end{bmatrix}$$

$$\check{\mathcal{H}}^{\kappa}_l(n, z) = \begin{bmatrix} \mathcal{H}^{\kappa}_{1,1,l}(n, z) & \cdots & \mathcal{H}^{\kappa}_{1,K,l}(n, z) \\ \mathcal{H}^{\kappa}_{2,1,l}(n, z) & \cdots & \mathcal{H}^{\kappa}_{2,K,l}(n, z) \\ \vdots & \cdots & \vdots \\ \mathcal{H}^{\kappa}_{K,1,l}(n, z) & \cdots & \mathcal{H}^{\kappa}_{K,K,l}(n, z) \end{bmatrix}$$

$$\mathcal{H}^{\kappa}_{i,j,l}(n, z) = \sum_{m=1}^{Q} \overline{\mathcal{H}}^{\kappa}_{i,j,l,m}(n, z).$$

Conditions for the existence of a filter $\mathcal{W}(n, z)$ were detailed in [11] and [12]. The output of the filter relative to the nth symbol relative to the kth user is denoted $z_k(n)$ and can be expressed as

$$\sum_{i=1}^{K} \sum_{l=1}^{Q} \sum_{\kappa=1}^{A} \sum_{m=L_1}^{L_2} w^{\kappa}_{k,i,l}(n, m) y^{\kappa}_{i,l}(n-m) = z_k(n). \tag{10}$$

III. Algorithmic Approach

Minor modifications of the derivations reported already in [11] and [15] (in fact time-varying cumulants have to be used due to the variations of the fading coefficients) give the super-exponential time-varying equations[2] (see also the Appendix)

$$\tilde{\mathbf{w}}^{[1]}_k(n) = \tilde{\mathbf{R}}(n)^{\dagger} \mathbf{D}_k(n) \tag{11}$$

$$\tilde{\mathbf{w}}^{[2]}_k(n) = \frac{\tilde{\mathbf{w}}^{[1]}_k(n)}{\sqrt{\tilde{\mathbf{w}}^{[1]^H}_k(n) \tilde{\mathbf{R}}(n) \tilde{\mathbf{w}}^{[1]}_k(n)}} \tag{12}$$

where $\tilde{\mathbf{w}}^{[1]}_k(n)$ and $\tilde{\mathbf{w}}^{[2]}_k(n)$ are the weights of the filter $\tilde{\mathbf{w}}_k(n)$

$$\tilde{\mathbf{w}}_k(n) = \left[\mathbf{w}^{1^T}_k(n), \mathbf{w}^{2^T}_k(n), \ldots, \mathbf{w}^{A^T}_k(n)\right]^T$$

$$\mathbf{w}^{\kappa}_k(n) = \left[\mathbf{w}^{\kappa,k^T}_{1,1}(n), \ldots, \mathbf{w}^{\kappa,k^T}_{K,Q}(n)\right]^T$$

$$\mathbf{w}^{\kappa,k}_{j,l}(n) = \left[w^{\kappa}_{k,j,l}(n, L_1), \ldots, w^{\kappa}_{k,j,l}(n, L_2)\right]^T$$

[2]The notation $\mathbf{A}^{\dagger}$ is used for the pseudoinverse of the matrix $\mathbf{A}$.

at time n obtained in the first and second step, respectively, of the iterative procedure, the matrix $\tilde{\mathbf{R}}(n)$ with dimensions $AQKL \times AQKL$ and the vector $\mathbf{D}_i(n)$ of fourth-order cumulants with dimensions $AQKL \times 1$ are given by

$$\tilde{\mathbf{R}}(n) = E\left\{\tilde{\mathbf{y}}(n)\tilde{\mathbf{y}}(n)^H\right\} \tag{13}$$

and

$$\tilde{\mathbf{D}}_i(n) = E\left\{\tilde{\mathbf{y}}^*(n)\tilde{z}_i(n)\right\}$$

where

$$\begin{aligned}\tilde{\mathbf{y}}(n) &= \left[\check{\mathbf{y}}^1(n)^T, \ldots, \check{\mathbf{y}}^A(n)^T\right]^T \\ \check{\mathbf{y}}^\kappa(n) &= \left[\mathbf{y}_{1,1}^\kappa(n)^T, \ldots, \mathbf{y}_{K,Q}^\kappa(n)^T\right]^T \\ \mathbf{y}_{i,l}^\kappa(n) &= \left[y_{i,l}^\kappa(n-L_1), \ldots, y_{i,l}^\kappa(n-L_2)\right]^T \\ \tilde{z}_i(n) &= \frac{\sigma_a^2}{\gamma_{4a}}\left(|z_i(n)|^2 - 2\sigma_a^2\right) z_i(n).\end{aligned}$$

Observe at this point that we cannot proceed as in [11] because time variations of the fading coefficients and as a consequence of the optimum distortionless reception filter do not allow the super-exponential iterations to achieve convergence. Ideally one should iterate several times (to obtain convergence) the super-exponential procedure at every time instant n: in other words one would need to "freeze" the evolution of the fading coefficients until convergence. This "naive" method prevents the application of a real-time algorithm where recursive operations are also iterations of the super-exponential algorithm, the strategy of the methods reported in [11] and [15]. We will attack the problem exploiting some ideas on basis expansions of fading processes studied in [4] and [16].

IV. Basis Expansion for the Time-Variant Filter

It was shown in [11] that the filter coefficients obtained from Step 1 of the procedure (11) and (12) are equivalently obtained minimizing the following cost function:

$$E\left\{\left|\sum_{i=1}^{K}\sum_{l=1}^{Q}\sum_{\kappa=1}^{A}\sum_{m=L_1}^{L_2} w_{k,i,l}^\kappa(n,m)y_{i,l}^\kappa(n-m) - \tilde{z}_k(n)\right|^2\right\} \tag{14}$$

or alternatively by solving an orthogonality-principle-like equation

$$E\left\{\left(\sum_{i=1}^{K}\sum_{l=1}^{Q}\sum_{\kappa_1=1}^{A}\sum_{m_1=L_1}^{L_2} w_{k,i,l}^{\kappa_1}(n,m)y_{i,l}^\kappa(n-m_1) - \tilde{z}_k(n)\right) \cdot y_{p,q}^{\kappa_2^*}(n-m_2)\right\} = 0 \tag{15}$$

for $\kappa_2 = 1, \ldots, A,\ p = 1, \ldots, K,\ q = 1, \ldots, Q,\ m_2 = L_1, \ldots, L_2$. These two expressions will be useful in the development that follows.

Consider the fading coefficents $f_{k,q}^\kappa(nT_s)$ and assume that their variations can be represented for a large enough $\mathcal{N}$ as

$$f_{k,q}^\kappa(nT_s) \simeq \sum_{p=1}^{\mathcal{N}} \overline{w}_{[f]}{}_{k,q,p}^{\kappa} c_p(n),$$

where $c_p(n)$ is a known basis (in [4] the basis is an exponential $c_p(n) = e^{j\omega_p n}$), and $\overline{w}_{[f]}{}_{k,q,p}^{\kappa}$ are the coefficients of the expansion. This representation is accurate under very general assumptions detailed in [4]. A number of interesting practical techniques can be developed because the method decouples the unknown time variations of the channel from the set of time-invariant coefficients. This method reveals its value in rapidly fading environments [4] where algorithms based on the typical quasi-static assumption of the fading channel usually fail.

We follow here a similar approach with the important difference that the basis expansion is performed on the time-variant distortionless reception filter $\mathcal{W}(n, z)$. Moreover we depart from the simple exponential basis to improve the representation accuracy.

A. A Wavelet Basis

As mentioned in Section I, we proceed to obtain a practical basis representation for the time variant filter $w_{i,k,l}^\kappa(n, m)$ applying a discrete wavelet transform (DWT) with respect to n over N available samples of the processes in (6) following an approach first proposed in [4] and [16].

One approach to implement the DWT is to use a binary subband tree structure that is constructed using stages of two-channel filterbanks [21]. Define $g_i(n)\ i = 0, 1$ as a dyadic perfect reconstruction filter bank [21] [in the z-domain $G_i(z)$]. The wavelet coefficients at resolution $P + 1 = 2$ for $w_{i_1,i_2,i_3}^\kappa(n, k)$ are given by [21]

$$\zeta_{1,n,i_1,i_2,i_3}(k) = \sum_l g_0(l)w_{i_1,i_2,i_3}^\kappa(2m-l, k) \tag{16}$$

$$\xi_{1,n,i_1,i_2,i_3}^\kappa(k) = \sum_l g_1(l)w_{i_1,i_2,i_3}^\kappa(2m-l, k). \tag{17}$$

The inverse wavelet transform is obtained from

$$w_{i_1,i_2,i_3}^\kappa(n, k) = \sum_{m=0}^{N/2-1} \zeta_{1,m,i_1,i_2,i_3}^\kappa(k)g_0^{(1)}(2^P m - n) + \sum_{m=0}^{N/2-1} \xi_{1,m,i_1,i_2,i_3}^\kappa(k)g_1^{(1)}(2m - n). \tag{18}$$

This method can be applied recursively to obtain at generic resolution depth P [20]

$$w_{i,i_1,i_2}^\kappa(n, k) = \sum_{m=0}^{N/2^P-1} \zeta_{P,m,i,i_1,i_2}^\kappa(k)g_0^{(P)}(2^P m - n) + \sum_{l=1}^{P}\sum_{m=0}^{N/2^l-1} \xi_{l,m,i,i_1,i_2}^\kappa(k)g_1^{(l)}(2^l m - n) \tag{19}$$

where $\zeta_{P,m,i,i_1,i_2}^\kappa(k)$ and $\xi_{l,m,i,i_1,i_2}^\kappa(k)$ are discrete wavelet transform coefficients and the filters $g_0^{(P)}(n)$ and $g_1^{(l)}(n)$ (with real coefficients) are given in the z-domain by $G_0^{(P)}(z) = \prod_{i=1}^{P-1} G_0(z^{2^i})$ and $G_1^{(l)}(z) = G_1(z^{2^l-1})\prod_{i=1}^{l-2} G_0(z^{2^i})$. Using vector notation it is possible to express (19) simply as

$$w_{i,i_1,i_2}^\kappa(n, k) = \mathbf{g}(P, n)^T \mathbf{w}_{[\mathbf{g}]}{}_{i,i_1,i_2}^{\kappa}(P, k) \tag{20}$$

where the organization of the wavelet coefficients in $\mathbf{w}_{[\mathbf{g}]_{i,i_1,i_2}^{\kappa}}(P, k)$ is

$$\mathbf{w}_{[\mathbf{g}]_{i,i_1,i_2}^{\kappa}}(P, k) = \left[\zeta_{P,i,i_1,i_2}^{\kappa}(k)^T \xi_{1,i,i_1,i_2}^{\kappa}(k)^T \cdots \xi_{P,i,i_1,i_2}^{\kappa}(k)^T\right]^T$$

with[3]

$$\begin{aligned}[\zeta_{P,i,i_1,i_2}(k)]_m &= \zeta_{P,m,i,i_1,i_2}^{\kappa}(k)\\ \left[\xi_{l,i,i_1,i_2}^{\kappa}(k)\right]_m &= \xi_{l,m,i,i_1,i_2}^{\kappa}(k).\end{aligned}$$

Evidently $\mathbf{g}(P, n) = [\mathbf{g}_0^{(P)}(n)^T, \mathbf{g}_1^{(1)}(n)^T, \ldots, \mathbf{g}_1^{(P)}(n)^T]^T$ where $[\mathbf{g}_i^{(l)}(n)]_m = g_i^{(l)}(2^l m - n)$ for $i = 0, 1$.

As it is, (19) is of little practical interest, however one can determine which components of the expansion (19) can be neglected (i.e., zeroed) without compromising the parsimony of the wavelet-based representation, and show in fact that it is possible to represent the time variations of the filter coefficients in a reduced order dimensional space. Indeed this is one of the celebrated merits of the wavelet transform [3]. In other words the excellent "compacting" properties (see [3]) of the wavelet transform are able to compress most of the energy of the time variations of the time-variant filter in the lower resolution representations of the processes and this makes the approach very attractive in practice. Defining $\overline{\mathbf{w}}_{[\mathbf{g}]_{i_1,i_2}^{i,\kappa}}(k)$, as the vector obtained zeroing the last $N - \mathcal{N} = \sum_{m=1}^{\mathcal{M}} N2^{-m}$ elements of $\mathbf{w}_{[\mathbf{g}]_{i,i_1,i_2}^{\kappa}}(k)$ and $\overline{\mathbf{c}}(n)$ as the vector obtained zeroing the last $N - \mathcal{N} = \sum_{m=1}^{\mathcal{M}} N2^{-m}$ elements of $\mathbf{g}(P, n)$, we can then write

$$w_{k,i_1,i_2}^{\kappa}(n, m) \simeq \overline{\mathbf{c}}(n)^T \overline{\mathbf{w}}_{[\mathbf{g}]_{i_1,i_2}^{\kappa,k}}(m). \tag{21}$$

It is possible to verify, for example, that for $N = 256$, speeds of the mobiles up to 300 km/h and $P = 3$ or $P = 4$ it is possible to impose $\xi_{l,m,i,i_1,i_2}^{\kappa}(k) = 0$ in (19) for any l, m, k without significant loss of representation accuracy. This means that we can have only $N/2^{\mathcal{M}} = N/2^P = 32$ or $N/2^{\mathcal{M}} = N/2^P = 16$ wavelet coefficients in $\overline{\mathbf{w}}_{[\mathbf{g}]_{i_1,i_2}^{\kappa,i}}(k)$.

B. Interpretations

The method we have outlined can be interpreted as a subspace selection procedure [19]. In fact the DWT of the N-long vector[4] $\mathbf{w}_{\mathbf{i}}^{\kappa}(k) = [w_{\mathbf{i}}^{\kappa}(1, k), w_{\mathbf{i}}^{\kappa}(2, k), \ldots, w_{\mathbf{i}}^{\kappa}(N, k)]^T$ representing the dynamics of the time-variant separating filter at lag k can be expressed as

$$\mathbf{w}_{\mathbf{i}}^{\kappa}(k) = \mathbf{W}\, \mathbf{w}_{[\mathbf{g}]_{\mathbf{i}}^{\kappa}}(P, k) \tag{22}$$

where $\mathbf{W}$ is an $N \times N$ orthonormal linear transformation expressing the operation of the wavelet transform. Consider $\overline{\mathbf{W}} = [\mathbf{W}_1, \ldots, \mathbf{W}_{\mathcal{N}}]^T$, the $N \times \mathcal{N}$ matrix formed by the first $\mathcal{N} = N2^{-\mathcal{M}}$ columns of $\mathbf{W}$ corresponding to the *shrinkage* (that is the zeroing) of the last $N - \mathcal{N}$ components of $\mathbf{w}_{[\mathbf{g}]_{\mathbf{i}}^{\kappa}}(P, k)$. Applying the trasformation

$$\theta_{\mathbf{i}}^{\kappa}(k) = \overline{\mathbf{W}}^H \mathbf{w}_{\mathbf{i}}^{\kappa}(k) \tag{23}$$

[3]The notation $[\mathbf{v}]_k$ is used for the kth element of vector $\mathbf{v}$.

[4]We have compacted a generic set of indices in $\mathbf{i} = [i_1, i_2, i_3]$.

we essentially define a "subspace parameter" $\theta_{\mathbf{i}}^{\kappa}(k)$. The $\mathcal{N}$ orthonormal columns of $\overline{\mathbf{W}}$ span an $\mathcal{N}$-dimensional subspace defined $\mathcal{W}_{\mathcal{N}}$ such that $\mathcal{W}_{\mathcal{N}} \subseteq \mathcal{C}^N$ (the space of any complex vector of length N). Observe that when the transformation $\overline{\mathbf{W}}$ is selected with $\mathcal{N} < N$, any estimate (or representation) of $\theta_{\mathbf{i}}^{\kappa}(k)$ is insensitive to disturbance components of $\mathbf{w}_{\mathbf{i}}^{\kappa}(k)$ for $m = \mathcal{N}+1, \mathcal{N}+2, \ldots, N$ and the resulting estimator (or representation) has a lower variance than the full space estimator. Also, any traditional adaptive scheme designed to follow the variations of $w_{i_1,i_2,i_3}^{\kappa}(n, k)$ at any point n attempts a full-space estimation of the separating filter. In other words traditional schemes will generate point-by-point estimates spanning $\mathcal{C}^N$. The proposed representation method when incorporated in practical detectors will achieve a decreased variance of the estimate error. It is however important to emphasize that this advantage is obtained at the expense of estimation bias because in practice

$$\mathbf{w}_{\mathbf{i}}^{\kappa}(k) = \overline{\mathbf{W}}\theta_{\mathbf{i}}^{\kappa}(k) = \mathbf{P}_W \mathbf{w}_{\mathbf{i}}^{\kappa}(k) \tag{24}$$

with $\mathbf{P}_W = \overline{\mathbf{W}}\overline{\mathbf{W}}^H$ holds only approximately true and while the variance increases with the dimensions of the subspace (that is $\mathcal{N}$) the bias decreases [19]. Evidently the goal of selecting a transformation $\overline{\mathbf{W}}$ is to optimize the trade-off between bias, variance, and computational load.

C. Solution of the Time-Varying Super-Exponential Equations

We have shown that an accurate representation of the filter variations is given by (21), or, renaming the components of the vectors

$$\begin{aligned}\overline{\mathbf{c}}(n) &= [c_1(n), c_2(n), \ldots, c_{\mathcal{N}}(n)]^T\\ \overline{\mathbf{w}}_{[\mathbf{g}]_{k,i_1,i_2}^{\kappa,k}}(m) &= \left[\overline{w}_{[g]_{i_1,i_2}^{\kappa,k}}(m)_1, \ldots, \overline{w}_{[g]_{i_1,i_2}^{\kappa,k}}(m)_{\mathcal{N}}\right]^T\end{aligned}$$

also by

$$w_{k,i_1,i_2}^{\kappa}(n, m) \simeq \sum_{p=1}^{\mathcal{N}} \overline{w}_{[g]_{i_1,i_2}^{\kappa,k}}(m)_p c_p(n).$$

Then, we can evidently write (14) as

$$E\left\{\left|\sum_{i=1}^{K}\sum_{l=1}^{Q}\sum_{\kappa=1}^{A}\sum_{m=L_1}^{L_2}\sum_{p=1}^{\mathcal{N}} \overline{w}_{[g]_{i,l}^{\kappa,k}} \cdot (m)_p c_p(n) y_{i,l}^{\kappa}(n-m) - \tilde{z}_k(n)\right|^2\right\}.$$

If we denote $\tilde{y}_{i,l,p}^{\kappa}(n, m) = c_p(n) y_{i,l}^{\kappa}(n-m)$, then (15) can be expressed as

$$E\left\{\left(\sum_{i_1=1}^{K}\sum_{l_1=1}^{Q}\sum_{\kappa_1=1}^{A}\sum_{m_1=L_1}^{L_2}\sum_{p_1=1}^{\mathcal{N}} \overline{w}_{[g]_{i_1,l_1}^{\kappa_1,k}}(m_1)_{p_1} \cdot \tilde{y}_{i_1,l_1,p_1}^{\kappa}(n, m_1) - \tilde{z}_k(n)\right) \tilde{y}_{i_2,l_2,p_2}^{\kappa_2}(n, m_2)\right\} = 0 \tag{25}$$

for $\kappa_2 = 1, \ldots, A,\ i_2 = 1, \ldots, K,\ l_2 = 1, \ldots, Q,\ m_2 = L_1, \ldots, L_2,\ p_2 = 1, \ldots, \mathcal{N}$. It is easily shown that

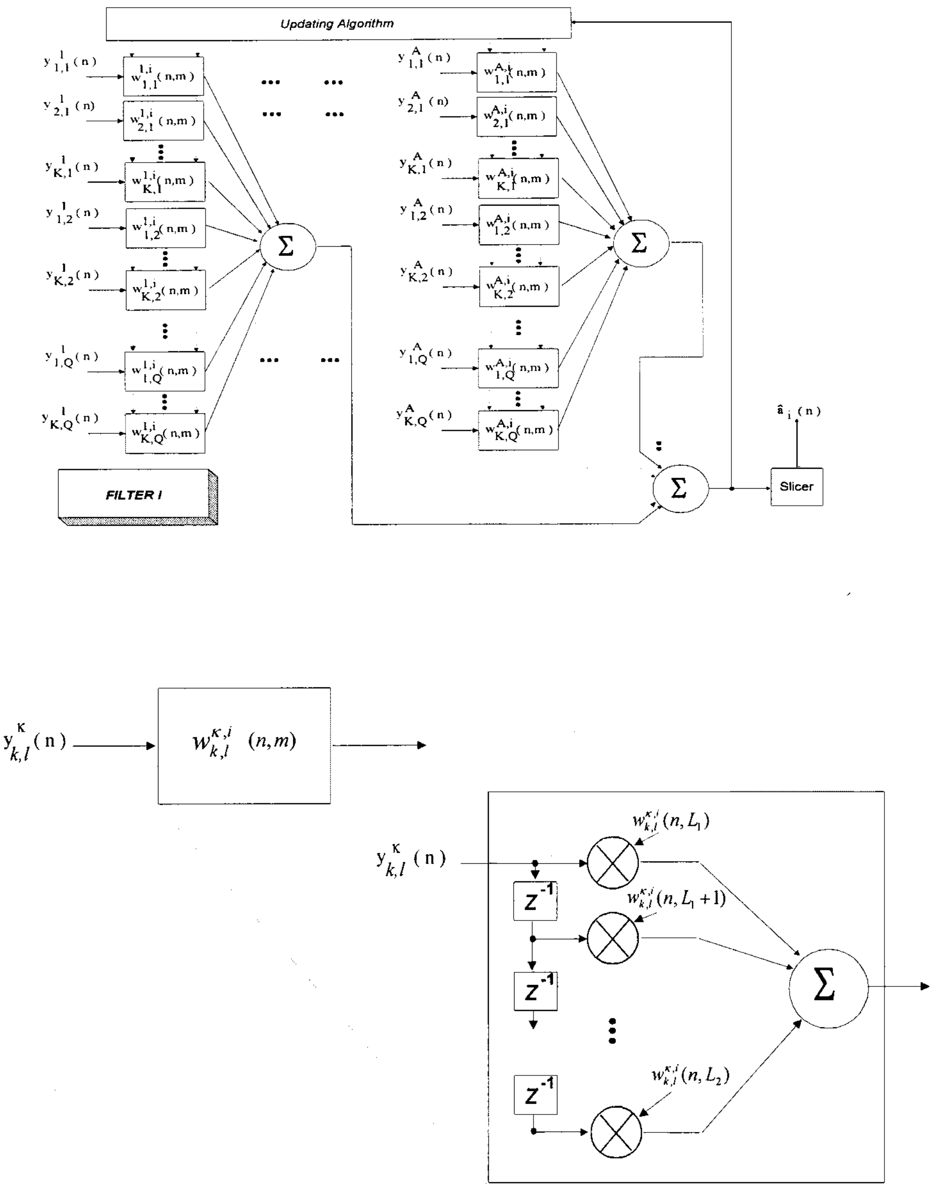

Fig. 4. The (time-varying) filter relative to the ith user.

the solution to the problem of obtaining $\overline{w}_{[g]^{\kappa_1,k}_{i_1,l_1}}(m_1)_{p_1}$, $p_1 = 1, \ldots, \mathcal{N}$, from (25) is given by

$$\overline{\mathbf{w}}_{[\mathbf{g}]_k} = \mathbf{R}^{\dagger}\mathbf{d}_k \tag{26}$$

where

$$\overline{\mathbf{w}}_{[\mathbf{g}]_k} = \left[\mathbf{w}_{[\mathbf{g}]_k^1}^{T}, \ldots, \mathbf{w}_{[\mathbf{g}]_k^A}^{T}\right]^T$$
$$\mathbf{w}_{[\mathbf{g}]_k^\kappa} = \left[\mathbf{w}_{[\mathbf{g}]^{\kappa,k}_{1,1}}^{T}, \ldots, \mathbf{w}_{[\mathbf{g}]^{\kappa,k}_{K,Q}}^{T}\right]^T$$
$$\mathbf{w}_{[\mathbf{g}]^{\kappa,k}_{j,l}} = \left[\overline{w}_{[g]^{\kappa,k}_{j,l}}(L_1)_1, \ldots, \overline{w}_{[g]^{\kappa,k}_{j,l}}(L_2)_{\mathcal{N}}\right]^T$$

$$\mathbf{R} = \left\langle \tilde{\mathbf{y}}_c^*(n)\tilde{\mathbf{y}}_c(n)^T \right\rangle_N \tag{27}$$
$$\mathbf{d}_k = \left\langle \tilde{\mathbf{y}}_c^*(n)\tilde{z}_k(n) \right\rangle_N \tag{28}$$

and we have used the notation $\langle x_n \rangle_N = (1/N)\sum_{n=1}^{N} x_n$ for an N-sample time average with

$$\tilde{\mathbf{y}}_{\mathbf{c}}(n) = \left[\check{\mathbf{y}}_c^1(n)^T, \ldots, \check{\mathbf{y}}_{\mathbf{c}}^A(n)^T\right]^T$$
$$\check{\mathbf{y}}_c^\kappa(n) = \left[\mathbf{y}_{\mathbf{c}^\kappa_{1,1}}(n)^T, \ldots, \mathbf{y}_{\mathbf{c}^\kappa_{K,Q}}(n)^T\right]^T$$
$$\mathbf{y}_{\mathbf{c}^\kappa_{i,l}}(n) = \left[\tilde{y}^\kappa_{i,l,1}(n, L_1), \ldots, \tilde{y}^\kappa_{i,l,\mathcal{N}}(n, L_2)\right]^T.$$

Observe that we used a technique similar to [5] where time-varying moments of the form

$$E\left\{\tilde{y}^{\kappa_1}_{i_1,l_1,p_1}(n, m_1)\tilde{y}^{\kappa_2}_{i_2,l_2,p_2}(n, m_2)^*\right\}$$

and

$$E\left\{\tilde{y}^{\kappa_1}_{i_1,l_1,p_1}(n, m_1)^*\tilde{z}_k(n)\right\}$$

are estimated by means of instantaneous products

$$\tilde{y}^{\kappa_1}_{i_1,l_1,p_1}(n, m_1)\tilde{y}^{\kappa_2}_{i_2,l_2,p_2}(n, m_2)^*$$

and

$$\tilde{y}^{\kappa_1}_{i_1, l_1, p_1}(n, m_1)^* \tilde{z}_k(n)$$

respectively, which are then collected for $n = 1, 2, \ldots, N$ and arranged to form an overdetermined system of linear equations in $\overline{\mathbf{w}}_{[\mathbf{g}]_k}$. Observe also that the first step of the super-exponential algorithm becomes

$$\tilde{\mathbf{w}}_k^{[1]}(n) = \overline{\mathbf{c}}(n)^T \mathbf{R}^\dagger \mathbf{d}_k \tag{29}$$

while the second step can be in practice performed by scaling $\tilde{\mathbf{w}}_k^{[1]}(n)$ [11] [15].

V. Adaptive Implementations

Typical approaches to achieve real-time implementation for (29) are gradient-based methods and recursive least qquares-based (RLS) methods. The well-known recursion in both cases is

$$\overline{\mathbf{w}}_{[\mathbf{g}]_k}(n+1) = \overline{\mathbf{w}}_{[\mathbf{g}]_k}(n) + \mathbf{p}_n \left[\tilde{\mathbf{y}}_{\mathbf{c}}(n), \overline{\mathbf{w}}_{[\mathbf{g}]_k}(n)\right] \tag{30}$$

where

$$\mathbf{p}_n \left[\tilde{\mathbf{y}}_{\mathbf{c}}(n), \overline{\mathbf{w}}_{[\mathbf{g}]_k}(n)\right] = \mu \left(\tilde{z}_k(n) - \tilde{\mathbf{y}}_{\mathbf{c}}(n)^T \overline{\mathbf{w}}_{[\mathbf{g}]_k}(n)\right) \tilde{\mathbf{y}}^*_{\mathbf{c}}(n)$$

if the algorithm of choice is the LMS and μ is a step-size parameter that controls the rate of adjustment. Using an RLS approach we have $\mathbf{p}_n[\tilde{\mathbf{y}}_{\mathbf{c}}(n), \overline{\mathbf{w}}_{[\mathbf{g}]_k}(n)] = \lambda \mathbf{K}^*_n \epsilon_n$ where $\epsilon_n = \tilde{z}_k(n) - \tilde{\mathbf{y}}_{\mathbf{c}}(n)^T \overline{\mathbf{w}}_{[\mathbf{g}]_k}(n)$, $\mathbf{K}_n = (\mathbf{P}_{n-1}\mathbf{a}_n/\lambda + \tilde{\mathbf{y}}_{\mathbf{c}}(n)^H \mathbf{P}_{n-1} \tilde{\mathbf{y}}_{\mathbf{c}}(n)$ and $\mathbf{P}_n = \lambda^{-1}(\mathbf{P}_{n-1} - \tilde{\mathbf{y}}_{\mathbf{c}}(n)^H \mathbf{P}_{n-1}\mathbf{K}_n)$. λ is a forgetting factor that can be set approximately equal to one because in reality the basis coefficients are time-invariant and need only to be identified, not tracked. This advantage in fast fading environments is extremely valuable and in fact results in significant performance advantages.

VI. Simulations

Since an analytical approach to the performance of the outlined detector is beyond the scope and the length of this paper we present the results of a simulation analysis. Consider a mobile radio system when the uplink applies DS/CDMA with a gross bit rate equal to 48.5 kbit/s. The users' codes are known at the receiver as the base station has allocated one code for each user. We use Gold codes of 15 chips, 7 equipower users and $Q = 2$ (two-path Rayleigh fading channel) in each channel impulse response between each user and each antenna array element. The antenna is a three-element uniform linear array with 3/4 of a wavelength spacing (this spacing guarantees uncorrelated scattering at the different antennas). The impulse responses of the multipath channels are generated so that the delays are constrained to be an integer number of chip periods according to Table I. Users modulate data using QPSK. A synchronization unit is assumed to estimate the delays exactly. We assume the first user to be the reference user. The signal-to-noise ratio (SNR) in the figures is equal to E_1^s/N_0. A training sequence of 14 symbols is transmitted every 256 symbols, to train the time-variant filter. During training the filter uses the known sequence instead of $\tilde{z}_k(n)$ in the adaptive scheme (the same approach was introduced in [11] to solve the

TABLE I
Channel Propagation Environments for Performance Evaluation Results: Delay Spreads of the Mobiles

User	$\tau^1_{k,1}, \tau^1_{k,2}$	$\tau^2_{k,1}, \tau^2_{k,2}$	$\tau^3_{k,1}, \tau^3_{k,2}$	Speed
1	$0T_c, 2T_c$	$1T_c, 2T_c$	$1T_c, 4T_c$	100 Km/hr
2	$1T_c, 3T_c$	$0T_c, 2T_c$	$3T_c, 4T_c$	80 Km/hr
3	$0T_c, 4T_c$	$1T_c, 5T_c$	$3T_c, 4T_c$	50 Km/hr
4	$1T_c, 2T_c$	$2T_c, 3T_c$	$3T_c, 4T_c$	10 Km/hr
5	$1T_c, 1T_c$	$2T_c, 4T_c$	$3T_c, 4T_c$	20 Km/hr
6	$2T_c, 4T_c$	$1T_c, 3T_c$	$3T_c, 4T_c$	80 Km/hr
7	$1T_c, 5T_c$	$2T_c, 3T_c$	$3T_c, 4T_c$	100 Km/hr

start-up problem). The slots are generated of dyadic length and $G_0(z)$ $G_1(z)$ are Daubechies filters of order 3. The Doppler frequency describes the second order statistics of channel variations. Doppler frequency is related through wavelength $\tilde{\lambda}$ to the ith mobile transmitter velocity V_i expressed in kilometers/hour (km/h). The model used in this case is based on the wide-sense stationary uncorrelated scattering (WSSUS) assumption. The complex weights are generated as filtered Gaussian processes fully specified by the scattering function. Particularly each process has a frequency response equal to the square-root of the Doppler Power Density Spectrum.[5] It is then straightforward to verify that for speeds of the mobile up to 300 km/h and $P = 4$ or $P = 5$ it is possible to retain only $N/2^{\mathcal{M}} = 32$ or $N/2^{\mathcal{M}} = 16$ wavelet coefficients in $\mathbf{w}^{\kappa}_{i_1, i_2, i_3}(P, k)$. Fig. 5 shows experiments for a time-varying multipath at 100 km/h, in a single user environment (user 1 in Table I). The DWT super-exponential filter uses a decomposition of a 500 samples (symbol spaced) snapshot of the received signal. Dashed traces represent the real part of the highest energy tap of the optimum separating filter computed as described in [11] at any time step assuming perfect channel information. Solid traces are obtained using a $P = 5$ DWT decomposition for the real part of the tap dynamics (top figure) and the traditional MMSE-QR-RLS (see below for the description of the method). The excellent "compacting" properties (see [3]) of the wavelet transform are able to compress most of the energy of the time variations of the channel in the low resolution representation in fact we retain only 16 wavelet coefficients. Bit-error rate (BER) analysis results are shown in Figs. 6 and 7. We compare with the multiuser MMSE detector updated using the QR-RLS algorithm (probably the best method in terms of tracking performance). The MMSE-QR-RLS at every iteration solves the following minimization problem:

$$\min_{\mathbf{g}_k} \left\| \begin{bmatrix} \lambda \mathbf{Y}(n) \\ \tilde{\mathbf{y}}(n+1)^T \end{bmatrix} \mathbf{g}_k - \begin{bmatrix} \lambda \mathbf{a}_k(n) \\ a_k(n+1) \end{bmatrix} \right\|^2 \tag{31}$$

where $\mathbf{Y}(n)$ and $\mathbf{a}(n)$ are recursively defined as

$$\mathbf{Y}(n) = \begin{bmatrix} \lambda \mathbf{Y}(n-1) \\ \tilde{\mathbf{y}}(n)^T \end{bmatrix}, \quad \mathbf{a}_k(n) = \begin{bmatrix} \lambda \mathbf{a}_k(n-1) \\ a_k(n) \end{bmatrix}$$

[5]The Doppler spectrum is approximated by rational filtered processes. The filters are described by their 3-dB bandwidth which is called the normalized Doppler frequency. The additional assumption is that all channels and complex weights have the same Doppler spectrum.

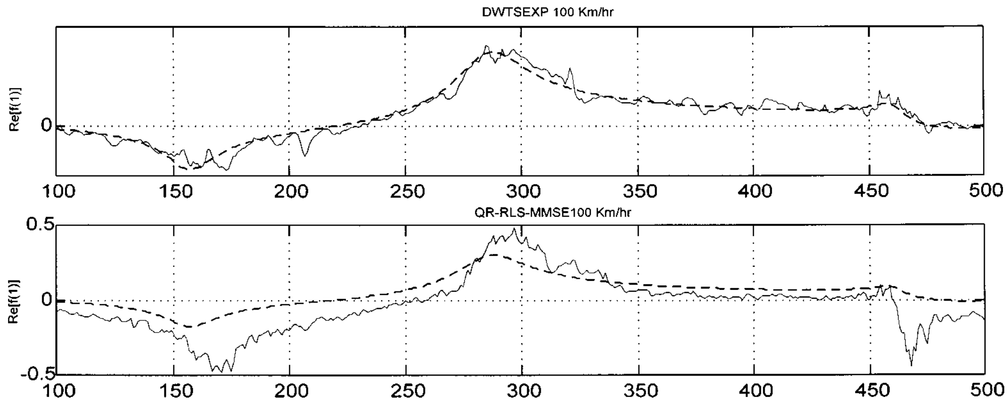

Fig. 5. Example of DWT-based tracking event for a 500-sample trace of the optimal separating filter (real part of the highest energy tap) as it compares with the QR-RLS tracking algorithm. Dashed lines denote the true trajectory of the optimum filter computed assuming perfect channel knowledge.

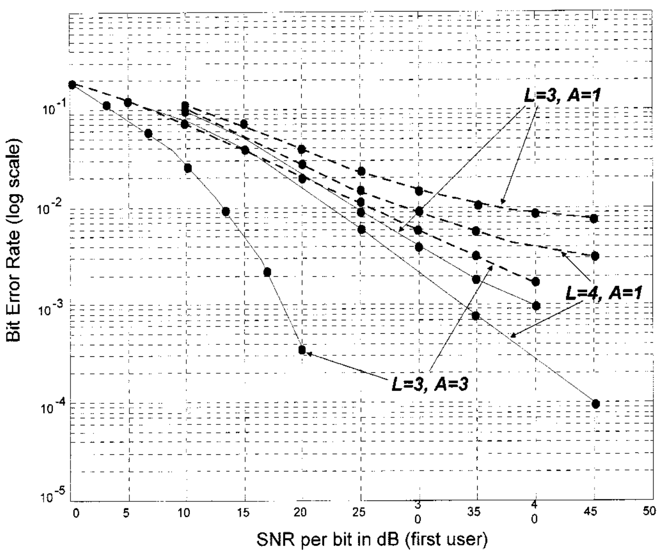

Fig. 6. BER for seven users, 15-length Gold codes in fast frequency-selective fading channels. For propagation parameters, see Table I. Solid curves denote the time-varying separating filter based on the DWT representation. Dashed curves denote the MMSE time-invariant filter based on the traditional adaptive QR-RLS scheme.

with

$$\tilde{\mathbf{y}}(n) = \left[\check{\mathbf{y}}^1(n)^T, \ldots, \check{\mathbf{y}}^A(n)^T\right]^T$$
$$\check{\mathbf{y}}^\kappa(n) = \left[\mathbf{y}^\kappa_{1,1}(n)^T, \ldots, \mathbf{y}^\kappa_{K,Q}(n)^T\right]^T$$
$$\mathbf{y}^\kappa_{i,l}(n) = \left[y^\kappa_{i,l}(n-L_1), \ldots, y^\kappa_{i,l}(n-L_2)\right]^T$$
$$\mathbf{g}_k = \left[\mathbf{g}_k^{1^T}, \ldots, \mathbf{g}_k^{A^T}\right]^T$$
$$\mathbf{g}_k^\kappa = \left[\mathbf{g}_{1,1}^{\kappa,k^T}, \ldots, \mathbf{g}_{K,Q}^{\kappa,k^T}\right]^T$$
$$\mathbf{g}_{j,l}^{\kappa,k} = \left[\overline{g}_{j,l}^{\kappa,k}(L_1), \ldots, \overline{g}_{j,l}^{\kappa,k}(L_2)\right]^T.$$

The solution obtained applying a recursive QR decomposition to the data matrix $\mathbf{Y}(n)$ defines the QR-RLS (QR decomposition based recursive least squares) [14] algorithm which attempts convergence to the MMSE solution for $\mathbf{g}_k$. We use in the DWT representation of the time-variant filter $P = 4$ and retain $N/2^{\mathcal{M}} = 32$ (for $N = 256$). BER is relative to the first mobile transmitter. Ideal frame and symbol synchronization is assumed and the LMS is used to update the wavelet coefficients. A sample size of 10^{k+4} was used to estimate an error probability of 10^{-k}. Fig. 6 shows results for the propagation environment reported in Table I and 1 Antenna compared to three-antenna reception. Evidently this scenario represents a fast fading environment, such as those practically found in high mobility cellular systems. $L = L_2 - L_1 + 1$ is the length of the filter, A is the number of sensors at the receiving antenna array. It is clear that the MMSE approach (dashed curves in Fig. 6) is inadequate basically because of the rapid variations of the channels. Fig. 7 shows the results of experiments with only the first three users of Table I active simultaneously, one single antenna, SNR per bit equal to 32 dB, and speed of the three mobile transmitters increased up to 400 km/h. This last experiment clearly emphasizes the advantages of the proposed method in extremely fast fading. The DWT-based filter is significantly less sensitive to Doppler spread increases.

VII. Conclusion

We have presented a new approach to suppress multiuser interference and channel distortions in a code-division multiple-access system when RF propagation between the users and the base-station receiver is distorted by arbitrarily time-varying multipath. The problem can be reconnected to a multichannel time-varying deconvolution problem when the channel is possibly non minimum-phase and the input distribution is non-Gaussian. The unknown optimum filter time variations are decomposed using optimal unconditional bases such as orthonormal wavelet bases and obtained by applying the super-exponential equations [11] and [15]. We focused on a multiscale time-variant adaptive filter characterized by the same dynamical features exhibited by the channel and we proved the approach extremely effective with respect to the traditional MMSE receiver. The fundamental contributions of the paper are summarized in these points as follows.

- A novel practical approach to multi-input multi-output (MIMO) blind time-variant deconvolution based on wavelet basis expansion was introduced.
- The application of the new method to the DS/CDMA detection problem was detailed.

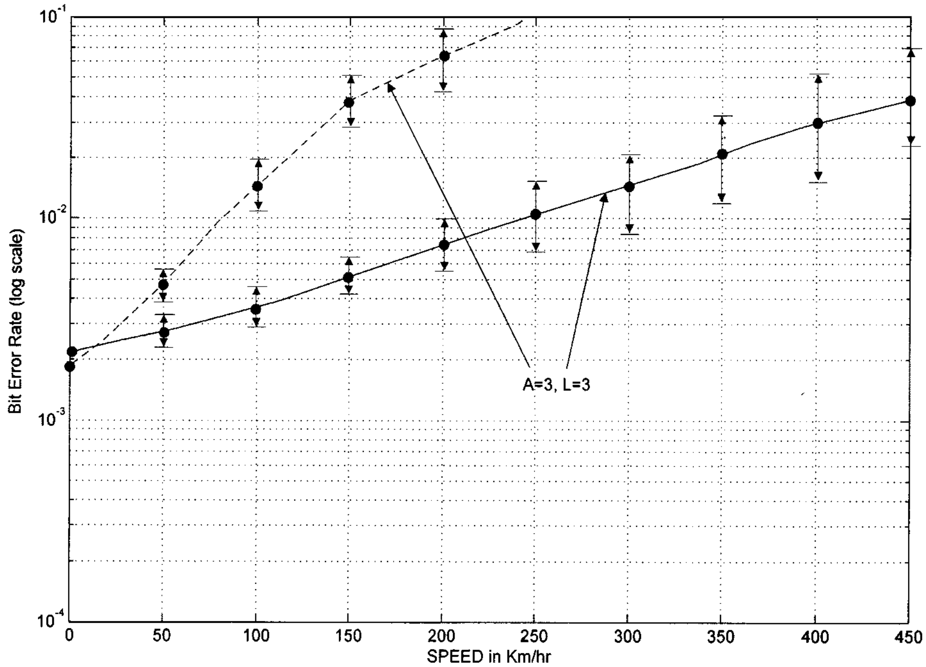

Fig. 7. BER for three users, 15-length Gold codes in fast frequency-selective fading channels with increasing speed of user 1, 2, 3. For propagation parameters, see Table I, first three users. One single antenna is used and the SNR per bit is equal to 32 dB. Solid curves denote the time-variant separating filter based on the DWT representation. Dashed curves denote the MMSE time-invariant filter based on the traditional adaptive QR-RLS scheme. The 90% confidence intervals for the measured BER are also shown.

- An important observation was made: multiscale decomposition imposes a subspace constraint to the dynamics of any deconvolution filter and appears to accelerate dramatically the convergence and tracking capability of any blind or nonblind adaptive algorithm.

Simulation results have been shown to demonstrate that the approach outperforms typical alogorithms (based on a quasi-static or slow-fading assumption) in time-varying channels characterized by Doppler spreads in excess of 50 Hz.

APPENDIX
TIME-VARYING CUMULANTS AND THE SUPER-EXPONENTIAL TIME-VARYING EQUATIONS

The $\mathcal{K}$th-order time-varying cumulant for the process (6) (in this paper $\mathcal{K} = 2$ or $\mathcal{K} = 4$) at lag $m_0, m_1, \ldots, m_{\mathcal{K}-1}$ is indicated as

$$c_{y^{(*)}_{i_1, l_1}, \ldots, y^{(*)}_{i_{\mathcal{K}}, l_{\mathcal{K}}}}(n;\, m_0, \ldots, m_{\mathcal{K}-1}) = \operatorname{cum}\left[y^{(*)}_{i_1, l_1}(n+m_0), \ldots, y^{(*)}_{i_{\mathcal{K}}, l_{\mathcal{K}}}(n+m_{\mathcal{K}-1})\right] \quad (32)$$

where $x^{(*)}$ denotes an optional complex conjugation of x. It is easily shown that a generalization of the equations shown in [11] (valid for time-invariant channels) can be obtained from the time-varying Bartlett equation (see [17]) (valid for $\mathcal{K} = 2$ only in the absence of noise)

$$c_{y^{(*)}_{i_1, l_1}, \ldots, y^{(*)}_{i_{\mathcal{K}}, l_{\mathcal{K}}}}(n;\, m_0, \ldots, m_{\mathcal{K}-1}) = \gamma_{\mathcal{K}a} \sum_{k=1}^{K} \sum_{m} \prod_{p=1}^{\mathcal{K}} h^{(*)}_{i_p, k, l_p}(n,\, m+m_{i-1}) \quad (33)$$

for $i_1, \ldots, i_{\mathcal{K}}$ chosen in the set $[1, 2, \ldots K]$, $l_1, \ldots, l_{\mathcal{K}}$ chosen in the set $[1, 2, \ldots Q]$. We do not report the detailed derivation of the time-varying super-exponential equations because they can be easily derived from equations already available in [11] and [15]. In fact evidently in the absence of noise

$$c_{y_{i_1, l_1}, y^*_{i_2, l_2}}(n;\, m_0, m_1) = \sigma_a^2 \sum_{k=1}^{K} \sum_{m} h_{i_1, k, l_1}(n,\, m+m_0) h^*_{i_2, k, l_2}(n,\, m+m_1)$$

and

$$c_{y^*_{i_1, l_1}, z_i, z_i^*, z_i}(n;\, m_0, 0, 0, 0) = \gamma_{4a} \sum_{k=1}^{K} \sum_{m} h^*_{i_1,k,l_1}(n, m+m_0) |s_{i,k}(n,m)|^2 s_{i,k}(n,m)$$

are equations analogous to (20) and (21) of [11], respectively, with

$$s_{i, j_1}(n,\, m_2) = \sum_{\kappa=1}^{A} \sum_{j=1}^{K} \sum_{l=1}^{Q} \sum_{m} w^{\kappa}_{i, j, l}(n,\, m) h^{\kappa}_{j, j_1, l}(n,\, m_2 - m), \quad \text{for } i, j_1 = 1, 2, \ldots, K.$$

ACKNOWLEDGMENT

The author would like to thank Prof. J. B. Anderson and Prof. G. J. Saulnier, Rensselaer Polytechnic Institute, and Dr. E. Montolivo, Fondazione Ugo Bordoni, for their support.

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